

Topological Sort (Kahn's BFS)

● MEDIUM

■ Introduction

Kahn's Algorithm uses BFS and the concept of IN-DEGREE (number of incoming edges). Nodes with in-degree 0 have no prerequisites — they can be processed first. After processing, reduce neighbors' in-degree; new zero-degree nodes join the queue.

■ Core Intuition

Kahn's Steps

- Compute in-degree of every node.
- Add all nodes with in-degree = 0 to queue.
- While queue not empty: dequeue node, add to result, reduce in-degree of all neighbors.
- If neighbor's in-degree becomes 0 → add to queue.
- If result size != V at end → cycle exists (topological sort impossible).

■ Java Implementation

Java

```
class Solution {
    public int[] topoSort(int V, ArrayList<ArrayList<Integer>> adj) {
        int[] inDegree = new int[V];

        // Calculate in-degree of each node
        for (int i=0; i<V; i++)
            for (int neighbor : adj.get(i))
                inDegree[neighbor]++;

        // Add all nodes with in-degree 0 to queue
        Queue<Integer> queue = new LinkedList<>();
        for (int i=0; i<V; i++)
            if (inDegree[i] == 0) queue.add(i);

        int[] result = new int[V];
        int idx = 0;

        while (!queue.isEmpty()) {
```

```

int node = queue.poll();
result[idx++] = node;

for (int neighbor : adj.get(node)) {
    inDegree[neighbor]--;
    if (inDegree[neighbor] == 0) queue.add(neighbor);
}
}

// If idx != V, there was a cycle
return (idx == V) ? result : new int[0];
}
}

```

■ Time & Space Complexity

Case	Time	Space	Why
Time	$O(V + E)$	$O(V)$	In-degree calc $O(E)$, BFS $O(V+E)$
Space	$O(V)$	$O(V)$	In-degree array + queue both $O(V)$

Kahn's vs DFS Topo Sort

- Kahn's (BFS): naturally detects cycles (result size $< V$), iterative, intuitive.
- DFS Topo: needs separate cycle detection, recursive (stack overflow risk on large graphs).
- Both produce valid topological orders. Prefer Kahn's for cycle detection built-in.

■ Practice Problems

Case	Time	Space	Why
Medium	LC 207	Course Schedule	Check cycle using in-degree
Medium	LC 210	Course Schedule II	Return Kahn's topo order
Hard	LC 1136	Parallel Courses	Min semesters = max topo level